XVII.

CONTRIBUTIONS FROM THE PHYSICAL LABORATORY OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

XVI. EXPERIMENTS ON THE FATIGUE OF SMALL SPRUCE BEAMS.

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THE following experiments were undertaken with the object of determining if possible what part of the so-called breaking load of a beam would ultimately cause the beam to break, all the conditions being the most favorable.

Incidental to the experiments, the moduli of rupture and of elasticity of small beams of kiln-dried spruce were determined.

The experiments were made in the Physical Laboratory of the Massachusetts Institute of Technology, the testing-machine used being the same as that described in a paper by the writer presented to the Academy Feb. 9, 1881.

With this machine the loads are applied by suspending known weights directly from the centre of the beam. The deflections of the beams were measured by means of a micrometer screw, the principle of electrical contact being taken advantage of in reading it. The moduli given have been computed from deflections measured to thousandths of an inch or hundredths of a millimetre.

As the load was suspended from a bolt resting upon the beam at the centre, it was necessary to measure the deflections one inch from the centre. For the small deflections from which the moduli of elasticity were determined, the difference between the measured deflection and the actual deflection is so small that it would not come within the limit to which the deflection was measured. For the deflections given in the tables, the deflections at the centre would be somewhat larger, but the error does not practically affect the results.

As the room in which these experiments were made is kept very warm and dry, any unseasoned timber would be so affected by the

heat that it would be impossible to tell whether the deflections were caused entirely by the load, or partly by the heat of the room; hence it was thought best in making these experiments to use kiln-dried timber.

The small beams upon which the experiments were made were taken from two spruce planks, selected from lumber which had been cut in Maine during the previous season. The planks were kept in a drying-kiln three weeks, and were then cut up into pieces about two inches square and allowed to dry until tested. For convenience the beams cut from one plank are classed as Series No. 2, and those from the other as Series No. 3; Series No. 1 including those beams previously experimented upon, which were discussed in my previous paper.

All the pieces of wood experimented upon were what might almost be called perfect pieces, being straight grained and free from knots. They were about $1\frac{1}{2}$ inches square, and 40 inches between the supports. The exact dimensions, with other data, are shown in the tables.

TABLE I.

SERIES No. 1. UNSEASONED SPRUCE.

No. of test piece.	Clear span l.	Breadth B.	Depth D.	E.	R.	Centre breaking weight for beam, 1″ × 1″ × 1′. A.	Deflection just before breaking.
	in.	in.	in.	lbs.	lbs.	lbs.	in.
1	40	1.475	1.45	1,731,000	11,380	632	1.565
2	40	1.445	1.52	1,556,000	10,330	574	1.395
3	40	1.469	1.448	1,765,000	10,710	595	1.48*
4	40	1.42	1.498	1,736,000	10,830	601	1.466
5	40	1.45	1.485	1,688,000	11,980	665	1.579
6	40	1.48	1.44	1,795,000	11,040	613	....
7	40	1.464	1.46	1,682,000	10,570	587	....
8	40	1.42	1.48	1,647,000	11,280	626	1.571
9	40	1.46	1.46	1,704,000	11,180	621	1.425
10	40	1.441	1.46	1,616,000	12,440	691	1.81*

Average value of E, 1,692,000 lbs.
" " " R, 12,170 lbs., of A, 620 lbs.

* Approximately.

Tables I., II., and III. are so arranged that a comparison of the strength and stiffness, together with the ultimate deflection of the pieces in the different series, can easily be made.

TABLE II.

Series No. 2. Kiln-dried Spruce.

No. of test piece.	Clear span l.	Breadth B.	Depth D.	E.	Deflection just before breaking.	R.	Centre breaking weight for beam, 1″ × 1″ × 1′. A.
	in.	in.	in.	lbs.	in.	lbs.	lbs.
1	40	1.52	1.52	1,573,000	1.676	12,560	698
2	40	1.495	1.5	....	1.656	13,590	755
3	40	1.52	1.5	....	1.517	12,540	697
4	40	1.51	1.503	....	1.816	13,720	762
5	40	1.506	1.506	....	1.662	13,740	763
6	40	1.51	1.516	1,760,000	1.937	Broke under $\frac{3}{4}$ b. w.	
7	40	1.508	1.508	1,636,000	1.79	Broke under $\frac{2}{3}$ b. w.	
8	40	1.51	1.518	1,721,000	....	Carried $\frac{2}{3}$ b. w. 22 days.	
9	40	1.5	1.504	1,580,000	....	Tested with $\frac{1}{2}$ b. w.	

Average value of E for five pieces, 1,654,000 lbs.
" " " R " " " 13,230 "
" " " A " " " 735 "

The letter E is used to denote the modulus of elasticity in these tables, and R the modulus of rupture.

The quantity denoted by A is one eighteenth of the modulus of rupture.

It will be noted that the pieces in Series No. 1 were not kiln dried, but were taken from a plank selected from ordinary timber.

TABLE III.

Series No. 3. Kiln-dried Spruce.

No. of test piece.	Clear span l.	Breadth B.	Depth D.	Deflection just before breaking.	R.	Centre breaking weight for beam, 1″ × 1″ × 1′. A.
	in.	in.	in.	in.	lbs.	lbs.
1	40	1.54	1.535	1.59	10,500	583
2	40	1.54	1.54	1.654	10,596	588
3	40	1.545	1.54	1.638	10,644	591
4	40	1.54	1.545	1.42	8,487	471
5	40	1.54	1.54	1.575	9,200	511
6	40	1.54	1.532	1.607	Broke under $\frac{3}{4}$ b. w.	
7	40	1.54	1.54	1.567	Broke under $\frac{2}{3}$ b. w.	
8	40	1.541	1.541	....	Tested with $\frac{1}{2}$ b. w.	

Average value of R, for five pieces, 9,885 lbs.
" " " A, " " " 549 "

Series No. 2.

In commencing this series of experiments five of the beams were subjected to loads of 30 and 40 lbs., and the deflection measured at the end of one hour from the time the load was applied. From these deflections the moduli of elasticity have been calculated. The values given in Table II. are the average of the values obtained from the deflection under 30 lbs. and the deflection under 40 lbs.

Having determined the moduli of elasticity of these pieces, five pieces of the series were broken by means of a gradually increasing load, and from their breaking load the modulus of rupture of each piece was computed. The average value of these five pieces (Nos. 1–5) was then considered to be the average value for the whole series, and the breaking weight of the remaining pieces of the series was computed on this basis.

Before attempting to break the remaining pieces, a load of 50 lbs., about $\frac{1}{15}$ of its breaking load, was applied to piece No. 6, with the object of determining if the deflection under this slight load would continually increase. The load was kept on the beam 288 hours, and the deflections, taken at intervals, are given in Table IV. From these it will be seen that the deflection increased very rapidly for the first 24 hours, and then quite regularly, but slowly, for 192 hours, and that after that it continued to *decrease* for 72 hours, when it slightly increased again.

As it was desired to use the machine for the more direct purposes of the experiments, the piece was removed from the machine, but it would have been interesting to have watched the further action of the load on the beam.

During the time that the deflections *decreased*, the weather was very wet, and it is the opinion of the writer that the deflections were somewhat affected by the change in the condition of the atmosphere. It should be observed that the greatest increase of deflection was very small, being only 0.44 of a millimetre, or about 0.017 of an inch.

After allowing this same beam several days in which to recover from the strain caused by the load of 50 lbs., 574 lbs., or $\frac{3}{4}$ of its calculated breaking load, was suspended from the beam, and the deflection measured at frequent intervals, with the results shown in Table IV. After carrying the load 260 hours the beam broke.

TABLE IV.

Deflections of Piece No. 6, Series No. 2, under a continued Load.

Load of 50 lbs. — $6\frac{1}{2}$ per cent. of Breaking Weight.					
Time applied.	Deflection.	Time applied.	Deflection.	Time applied.	Deflection.
hours.	mm.	hours.	mm.	hours.	mm.
0	2.04	120	2.305	240	2.408
24	2.22	144	2.368	264	2.358
72	2.265	168	2.418	288	2.368
96	2.283	192	2.478	Load removed.	
Load of 574 lbs. or $\frac{3}{4}$ of Calculated Breaking Weight.					
Time applied.	Deflection.	Time applied.	Deflection.	Time applied.	Deflection.
hours.	mm.	hours.	mm.	hours.	mm.
0	25.51	69	33.83	140	38.17
1.5	28.72	75	34.33	165	39.83
4.5	30.58	92	35.28	188	40.58
19.5	31.72	117	37.17	237	41.59
				260*	42.47
* Broke shortly after.					

Piece No. 7 of this series was computed to hold 756 lbs. before breaking, and 504 lbs., or $\frac{2}{3}$ of the breaking weight, was suspended from the beam. After supporting this load 134 hours the beam broke.

TABLE V.

Deflections of Piece No. 7, Series No. 2, under 504 lbs. or $\frac{2}{3}$ of its Calculated Breaking Weight.

Time applied.	Deflection.	Time applied.	Deflection.	Time applied.	Deflection.
hours.	mm.	hours.	mm.	hours.	mm.
0	23.04	48	34.43	120	43.16
14	28.48	86	38.06	134	45.46
24	31.26	96	40.04	Broke soon after.	
38	33.16	110	41.64		

The deflections of the beam measured at frequent intervals are given in Table V.

Piece No. 8 of this series carried $\frac{2}{3}$ of its breaking weight 499 hours, with an increase in deflection of 7.64 millimetres (0.3 in.).

As the deflection was constantly increasing, and was already more than the deflection of Piece No. 7 when the load was first applied, it seems to the writer that the beam would undoubtedly have in time been broken by its load.

The deflection of this beam is given in Table VI.

TABLE VI.

DEFLECTIONS OF PIECE NO. 8, SERIES NO. 2, UNDER 511 LBS. OR $\frac{5}{8}$ OF ITS CALCULATED BREAKING WEIGHT.

Time applied.	Deflection.	Time applied.	Deflection.	Time applied.	Deflection.
hours.	mm.	hours.	mm.	hours.	mm.
0	21.98	211	26.86	403	29.15
44	23.07	235	27.09	427	29.37
68	25.45	259	27.82	451	29.48
92	25.78	283	28.14	475	29.53
116	25.94	308	28.53	499	29.62
140	26.20	332	28.81	Weight taken off.	
168	26.43	379	29.02		

The last piece in Series No. 2, Piece No. 9, was subjected to a load of $\frac{1}{2}$ of its breaking weight for 327 hours, during which time the deflection constantly increased from 16.39 mm. (0.644 in.) to 19.07 mm. (0.75 in.). The load was then removed and the "set" of the beam measured. This set gradually decreased as the beam recovered itself, until it was quite small, and probably the larger part of it was due to the indentation of the beam at the points of support, something which cannot well be prevented in a wooden beam. It will be seen from table VII., that each time the load was applied the beam deflected a little more than at the previous application of the load; also that the set increased much faster than the deflection.

This tends to prove that the continued application and removal of one half of the breaking weight of a beam will in a comparatively short time cause it to break.

TABLE VII.

Experiments on Piece No. 9, Series No. 2.

Deflection under 374 lbs. or $\frac{1}{2}$ of its Calculated Breaking Weight.

Time applied.	Deflection.	Time applied.	Deflection.	Time applied.	Deflection.
hours.	mm.	hours.	mm.	hours.	mm.
0	16.39	48	17.96	211	18.74
2	17.08	66	18.08	234	18.87
18	17.49	116	18.17	279	18.91
25	17.77	138	18.34	303	19.01
42	17.86	164	18.43	327*	19.07

* Load removed.

Recovery of the piece on removal of the above load after 327 hours application.

Time to recover.	Set.	Time to recover.	Set.	Time to recover.	Set.
hours.	mm.	hours.	mm.	hours.	mm.
0	2.41	8	1.73	48	1.32
2	1.94	24	1.46	74	1.30*
4	1.74	32	1.38		

* At least .5 mm. of this set was due to the indentation of the beam at the points of support.

After 21 days rest the beam was again put in the machine, and the same load of 374 lbs. was alternatively applied and taken off, with the following results: —

Weight.	Deflection on application of load.	Time applied.	Deflection.	Set.	Time to recover.	Set.
lbs.	mm.	hours.	mm.	mm.	hours.	mm.
374	16.62	26	18.22	1.45	16	.53
"	17.34	8	18.54	1.60	15	.66
"	17.52	$4\frac{1}{2}$	18.49	1.70	$15\frac{1}{2}$	.67
"	17.75	$9\frac{1}{2}$	18.83	1.90	$14\frac{1}{2}$	.97
"	17.95	$9\frac{1}{2}$	19.00	1.97	$14\frac{1}{2}$	1.08
"	18.10	48	19.56	2.68	24	1.48
"	18.38	$9\frac{1}{2}$	19.52	2.50	$14\frac{1}{2}$	1.47
"	18.38	$9\frac{1}{2}$	19.48	2.40	$14\frac{1}{2}$	1.53
"	18.58	9	19.73	2.60	15	1.54
"	18.70	48	20.35	3.15	9	1.67
"	19.15	24	20.86	3.40	15	1.75
"	19.55	24	22.02	4.30	24	2.26
"	20.12	24	21.86	4.20	9	3.97
"	21.85	756	26.80	7.70	24	5.61
"	24.90	105	27.16	7.40	24	5.70

Note. — The numbers in column 5 show the set of the beam immediately after the removal of the load, which was suspended from the beam during the number of hours given in column 3.

Series No. 3.

The results of the second series of experiments convinced the writer that a perfect and dry spruce beam would in time break under a load of only one half of its calculated breaking weight, but to make the results more certain a third series was undertaken, with the same object in view.

The pieces of wood tested in this series were to all appearance equally as perfect and dry as those in Series No. 2. Table III. gives the dimensions of the beams in this series, the moduli of rupture of the first five pieces, and the ultimate deflection of all the pieces.

The average value of the modulus of rupture of the first five pieces was taken as the basis from which the breaking weight of pieces Nos. 6, 7, and 8 were computed.

Piece No. 6 of this series was broken by a load of $\frac{3}{4}$ of its calculated breaking weight, 22 days after the load was applied. The deflections of this beam at various intervals during the 22 days are given in Table VIII.

TABLE VIII.

DEFLECTION OF PIECE NO. 6, SERIES NO. 3, UNDER A LOAD OF 399 LBS. OR $\frac{3}{4}$ OF ITS CALCULATED BREAKING WEIGHT.

Time applied.	Deflection.	Time applied.	Deflection.	Time applied.	Deflection.
days.	mm.	days.	mm.	days.	mm.
0	19.12	5.5	25.31	17.5	31.53
0.5	21.53	6.5	25.40	19.5	33.84
1.5	22.90	10.5	27.40	20.5	36.05
3.5	24.85	12.5	28.79	21.5	39.09
4.5	25.25	13.5	29.09	22*	40.82

* Broke within 12 hours.

The next piece of the series, No. 7, was subjected to a load of $\frac{3}{4}$ of its breaking weight, which it carried $24\frac{1}{2}$ days, and then gave way as the others had done.

The deflections are given in Table IX.

TABLE IX.

DEFLECTION OF PIECE NO. 7, SERIES NO. 3, UNDER A LOAD OF 401 LBS. OR $\frac{2}{3}$ OF ITS CALCULATED BREAKING WEIGHT.

Time applied.	Deflection.	Time applied.	Deflection.	Time applied.	Deflection.
days.	mm.	days.	mm.	days.	mm.
0	20.07	9.5	31.68	18.5	33.19
1.5	23.77	10.5	31.93	19.5	33.28
3.5	26.98	11.5	32.04	20.5	33.58
4.5	29.70	12.5	32.30	23	35.57
5.5	30.37	15.5	32.70	23.5	37.04
6.5	30 80	16.5	32.85	24.5*	39.80
8.5	31.40	17.5	33.07		

* Broke within 12 hours.

Having proved that $\frac{2}{3}$ of the so-called breaking weight of a beam is more than it will carry permanently, the next beam was subjected to only $\frac{1}{2}$ of its calculated breaking weight.

This load was kept on the beam 49 days, during which time the deflection increased from 13.4 mm. (0.527 in.) to 18.55 mm. (0.73 in.) It was then necessary to remove the beam from the machine, that the latter might be used for other tests. The "set" of the beam on the removal of the load was 4.35 mm. (0.171 in.).

Seven days after the load was removed it was again put on the beam, and allowed to remain 77 days, when it was again removed, that the beam might be put on a temporary frame and kept there, with the same load suspended from it, until it broke.

The "set" of the beam on the second removal was only 3.76 mm. (0.148 in.), being less than what it was after the first removal.

The deflections of the beam are given in Table X.

As this beam continued constantly to deflect, and as this increase in deflection is still going on, it seems to the writer that it must ultimately break under this load, for when the deflection reaches a certain limit it will, as is shown by the other pieces, rapidly increase until it breaks.

Observations on Tables I., II., *and* III. Comparing Tables II. and III., we find a great difference in the values of the moduli of rupture for the two sets of experiments, although the planks from which the pieces were cut were selected from the same lot of lumber and dried the same length of time.

TABLE X.

Deflection of Piece No. 8, Series No. 3, under a Load of 301 lbs. or ½ of its Calculated Breaking Weight.

Time applied.	Deflection.	Time applied.	Deflection.	Time applied.	Deflection.
days	mm.	days.	mm.	days.	mm.
0	13.40	14	16.77	25	17.14
1	15.03	15	16.93	26	17.17
3	15.51	17	16.89	27	17.41
4	15.58	18	16.89	29	17.51
6	16.34	19	16.97	31	17.59
8	16.52	20	17.07	33	17.76
10	16.53	21	17.14	38	17.97
11	16.66	22	17.14	45	18.45
12	16.83	24	17.10	49	18.55
13	16.79				

After taking the last deflection the load was removed from the beam, when the centre of the beam returned to within 4.35 mm. of its original position. After 7 days the load of 301 lbs. was again put on the beam, causing the following deflections: —

Time applied.	Deflection.	Time applied.	Deflection.	Time applied.	Deflection.
days.	mm.	days.	mm.	days.	mm.
0	13.20	23	15.70	59	16.84
1	14.25	38	16.18	63	16.95
3	14.61	43	16.43	66	17.05
5	14.90	47	16.50	68	17.11
10	15.25	48	16.52	71	17.15
13	15.37	53	16.64	77	17.32
18	15.73	54	16.70		

The only reason which the writer can give for the low value of R in the third series is that the plank was sawn from the outside of the tree. It will be noticed that the values of R ran very high for the pieces in Series No. 2, also that the average value of R for Series No. 1 is only about 8 per cent less than that for Series No. 2, while it is about 23 per cent greater than the average for Series No. 3.

This would lead one to infer that ordinarily dry lumber does not have its strength materially increased by being kiln dried.

Comparing Tables I. and II., we see that the average value of the modulus of elasticity for the beams of unseasoned spruce is fully as large as that for the kiln-dried spruce. The beams in Table I., though denoted as unseasoned, were fully as dry as timber which has been in an ordinary building three months, but it was not artificially dried.

If we compare the ultimate deflections of all the pieces with their moduli of rupture, we shall find as a rule that those beams which were the strongest bent the most before breaking.

The values of E in Tables I., II., and III. were computed from the expression $E = \frac{Wl^3}{4 \Delta BD^3}$, Δ denoting the deflection in inches. The values of R were computed from the formula $R = \frac{3}{2} \frac{Wl}{BD^2}$.

From further observations of the tables we shall see that the deflections of Pieces Nos. 6 and 7 of Series No. 3 increased 100 per cent; or the deflection when the load was applied was only about $\frac{1}{2}$ what it was when the beam broke.

Also that the deflection of Piece No. 9, Series No. 2, and of Piece No. 8, Series No. 3, is much less than one half of what the ultimate deflection would probably be.

Hence I think it perfectly safe to conclude that for spruce-beams of small section a load which will produce a deflection of one half the maximum deflection of the beam before breaking will ultimately break the beam.

From a study of Tables VII. and X. it appears that a load of one half the so-called breaking load of a beam does not injure the beam when applied only for a short time; for it will be noticed that for both Pieces No. 9, Series No. 2, and Piece No. 8, Series No. 3, the deflection of the beam upon the second application of the load was almost the same as upon the first application, the difference being very slight indeed.

Effect of the "Annual Rings" on the Strength of a Beam.

After computing the moduli of rupture for the first five pieces of Series No. 2, the writer was surprised to see that three pieces had nearly the same modulus, and that the remaining two pieces also agreed almost exactly, but that there was a great difference between the moduli of the three and of the two pieces.

The writer could think of no reason for this phenomenon until he examined the fractured section of the beams, when it was discovered that in the three beams which had the high moduli the "annual rings" were parallel, or nearly so, with the top and bottom surfaces of the beam, while in the other two the "annual rings" made an angle of about 45° with these surfaces.

Conclusions.

The conclusions which may be drawn from the research here described, the writer considers to be as follows: —

That for spruce beams of small section, selected from lumber which has been moderately well seasoned and dried, the strength is not materially increased by the timber being kiln dried; that the modulus of elasticity is not proportional to the modulus of rupture; and that the elasticity is not increased by kiln-drying the timber.

That with small spruce beams those which have the greatest strength bend the most before breaking.

That when a load between $\frac{1}{2}$ and $\frac{7}{8}$ of the so-called breaking weight is applied to a small spruce beam it produces a deflection which for a few hours rapidly increases, until the beam has fairly settled under its load; from this time the deflection increases gradually until a short time before breaking, when it increases more and more rapidly.

That a load of $\frac{1}{2}$ of the so-called breaking weight if applied but for a few days does not injure such beams.

That a load which will cause such a beam to deflect one half of its maximum deflection before breaking will ultimately break the beam.

That under the most perfect conditions small spruce beams will not permanently support a load of one half their so-called breaking weight.

That the position of the annular rings in spruce beams of small section materially affects the strength of the beams, their strength being the least when the rings make an angle of 45° with the top and bottom surfaces of the beam.

The writer agrees with Prof. R. H. Thurston in considering 5 as the least factor of safety which should be used for wooden beams under an absolutely static load.